NAG Toolbox for MATLAB

f07md

1 Purpose

f07md computes the Bunch–Kaufman factorization of a real symmetric indefinite matrix.

2 Syntax

```
[a, ipiv, info] = f07md(uplo, a, 'n', n)
```

3 Description

f07md factorizes a real symmetric matrix A, using the Bunch–Kaufman diagonal pivoting method. A is factorized as either $A = PUDU^{\mathrm{T}}P^{\mathrm{T}}$ if **uplo** = 'U' or $A = PLDL^{\mathrm{T}}P^{\mathrm{T}}$ if **uplo** = 'L', where P is a permutation matrix, U (or L) is a unit upper (or lower) triangular matrix and D is a symmetric block diagonal matrix with 1 by 1 and 2 by 2 diagonal blocks; U (or L) has 2 by 2 unit diagonal blocks corresponding to the 2 by 2 blocks of D. Row and column interchanges are performed to ensure numerical stability while preserving symmetry.

This method is suitable for symmetric matrices which are not known to be positive-definite. If A is in fact positive-definite, no interchanges are performed and no 2 by 2 blocks occur in D.

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo – string**

Indicates whether the upper or lower triangular part of A is stored and how A is to be factorized.

uplo = 'U'

The upper triangular part of A is stored and A is factorized as $PUDU^{\mathrm{T}}P^{\mathrm{T}}$, where U is upper triangular.

uplo = 'L'

The lower triangular part of A is stored and A is factorized as $PLDL^{\mathrm{T}}P^{\mathrm{T}}$, where L is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **a**(**lda**,$*$) **– double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The n by n symmetric indefinite matrix A.

If **uplo** = 'U', the upper triangular part of A must be stored and the elements of the array below the diagonal are not referenced.

If **uplo** = 'L', the lower triangular part of A must be stored and the elements of the array above the diagonal are not referenced.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The second dimension of the array **a**.

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, work, lwork

5.4 Output Parameters

1: **a(lda,$*$) – double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The upper or lower triangle of A contains details of the block diagonal matrix D and the multipliers used to obtain the factor U or L as specified by **uplo**.

2: **ipiv($*$) – int32 array**

Note: the dimension of the array **ipiv** must be at least $\max(1, \mathbf{n})$.

Details of the interchanges and the block structure of D. More precisely,

if $\mathbf{ipiv}(i) = k > 0$, d_{ii} is a 1 by 1 pivot block and the ith row and column of A were interchanged with the kth row and column;

if **uplo** = 'U' and $\mathbf{ipiv}(i-1) = \mathbf{ipiv}(i) = -l < 0$, $\begin{pmatrix} d_{i-1,i-1} & \bar{d}_{i,i-1} \\ \bar{d}_{i,i-1} & d_{ii} \end{pmatrix}$ is a 2 by 2 pivot block and the $(i-1)$th row and column of A were interchanged with the lth row and column;

if **uplo** = 'L' and $\mathbf{ipiv}(i) = \mathbf{ipiv}(i+1) = -m < 0$, $\begin{pmatrix} d_{ii} & d_{i+1,i} \\ d_{i+1,i} & d_{i+1,i+1} \end{pmatrix}$ is a 2 by 2 pivot block and the $(i+1)$th row and column of A were interchanged with the mth row and column.

3: **info – int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **n**, 3: **a**, 4: **lda**, 5: **ipiv**, 6: **work**, 7: **lwork**, 8: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If **info** $= i$, $d(i,i)$ is exactly zero. The factorization has been completed, but the block diagonal matrix D is exactly singular, and division by zero will occur if it is used to solve a system of equations.

7 Accuracy

If **uplo** = 'U', the computed factors U and D are the exact factors of a perturbed matrix $A + E$, where

$$|E| \leq c(n)\epsilon P|U||D|\left|U^{\mathrm{T}}\right|P^{\mathrm{T}},$$

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

If **uplo** = 'L', a similar statement holds for the computed factors L and D.

8 Further Comments

The elements of D overwrite the corresponding elements of A; if D has 2 by 2 blocks, only the upper or lower triangle is stored, as specified by **uplo**.

The unit diagonal elements of U or L and the 2 by 2 unit diagonal blocks are not stored. The remaining elements of U or L are stored in the corresponding columns of the array **a**, but additional row interchanges must be applied to recover U or L explicitly (this is seldom necessary). If $\mathbf{ipiv}(i) = i$, for $i = 1, 2, \ldots, n$ (as is the case when A is positive-definite), then U or L is stored explicitly (except for its unit diagonal elements which are equal to 1).

The total number of floating-point operations is approximately $\frac{1}{3}n^3$.

A call to f07md may be followed by calls to the functions:

f07me to solve $AX = B$;

f07mg to estimate the condition number of A;

f07mj to compute the inverse of A.

The complex analogues of this function are f07mr for Hermitian matrices and f07nr for symmetric matrices.

9 Example

```
uplo = 'L';
a = [2.07, 0, 0, 0;
     3.87, -0.21, 0, 0;
     4.2, 1.87, 1.15, 0;
     -1.15, 0.63, 2.06, -1.81];
[aOut, ipiv, info] = f07md(uplo, a)
```

```
aOut =
    2.0700         0         0         0
    4.2000    1.1500         0         0
    0.2230    0.8115   -2.5907         0
    0.6537   -0.5960    0.3031    0.4074
ipiv =
          -3
          -3
           3
           4
info =
           0
```